Snakes on a Plain

Use pi to find the length of rivers that wind their way across the plains.

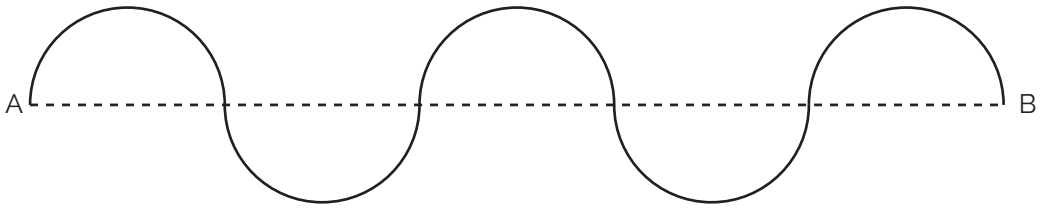


Figure 1: The Five-Snake River

Introduction

In my dream last night, I was trapped in an aluminum cylinder that was hovering precariously five miles above the Earth's surface. The only thing I had to eat was a tiny bag of five peanuts. Every couple of minutes the cylinder would shake unpredictably, thus reminding me of my delicate position and the terrifying demise that would result if the cylinder were to stop hovering. Oh, wait, that wasn't my dream; it was my flight back from Wichita.

I'm not a big fan of flying. I understand the physics of flight, and I am aware that I am much more likely to die in a bathtub accident or at the hands of a shark than I am to die in a plane crash, which is amazing because sharks don't even have hands. (I think the difference in probability is even more profound if the shark is actually IN your bathtub.)

Even though I know all this, I am still unnerved by planes. The whole thing seems unnatural to me. If God had intended for us to fly, he or she would have a left us a note letting us know. "Dear, humans, you can fly. I am leaving you this note to let you know because otherwise it might not be so apparent to you… what with the whole gravity thing and all…plus your lack of wings. Also, sorry about the mosquitoes. I don't know what I was thinking."

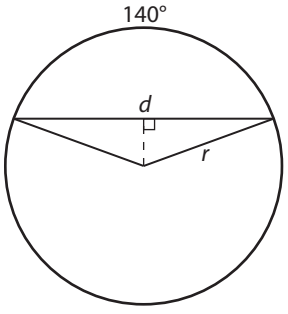
So, because I don't like flying, I look for ways to distract myself. One of my favorites is to look out the window and try to match geographic features with what I can see in the atlas I brought along for this purpose. While doing this on my flight from Wichita, I became fascinated by how the rivers always get from point A to point B in a back-and-forth snaking manner. If we saw a man doing this, we would stare and perhaps contact the

Fun River Facts

- ✓ There are 3,500,000 miles of rivers in the United States. Here's the surprise: You co-own a large percentage of these rivers because they are public domain up to their high-water marks.
- ✓ There are 18 countries that don't have a single river.
- ✓ Russia is the country with the most rivers. They have about 100,000!

Itty-Bitty Hint

- ✓ In Challenge 6, you'll need to relate the length of the arc to the length of the chord. To do that, you'll first need to relate the length of the chord to the radius of the circle of which the arc is a part.



authorities, but with rivers we understand that it is just a natural consequence of how they are formed.

Challenge 1

1. Figure 1 on the previous page shows a meandering river that has been modeled by five semi-circles. The teacher will give you a value for *AB*. Use this straight-line distance to find the length of the five-snake river.
2. Hmmm, I wondered to myself on the Wichita flight, how will this river length change if there are more congruent semi-circles? Using the same value for *AB*, calculate the length of the Ten-Snake River and the Two-Snake River.
3. Have the teacher initial your work.

Challenge 2

4. Algebra can be a powerful tool when analyzing problems like this one. Let n = the number of semi-circles, AB = the straight-line distance from A to B, and L = the total length of the river. Derive a formula for calculating L from the other variables.
5. If you did this correctly, you should get a result that confirms what you discovered in the first challenge.
6. Have your teacher initial your work.

Challenge 3

Let's turn up the level of fun. Suppose the river is like the one below. It is made up of five non-congruent semicircles with diameters as shown.

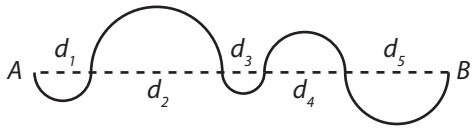


Figure 2: The Wiggly River

7. Make a diagram of your wiggly river. Label your five diameters with values that add up to *AB*.
8. Calculate the length of the river, and compare it to your answers from *Challenge 1*.
9. Have your teacher initial your work.

Challenge 4

It's now time for the grand finale of this part of the project. Imagine splitting the river up into *five* non-congruent semicircles. Let the diameters of the semicircles be d_1, d_2,...d_n. Note that these diameters add up to the length of the river as shown by this equation:

$$D = d_1 + d_2 + d_3 + ... + d_n$$

10. Derive a formula for calculating the total length of the river, L. Simplify this formula as much as you can. The distributive property is your friend.
11. How does this compare to your previous results using congruent semicircles?
12. Explain whether your discovery would or would notused 100 non-congruent circles.
13. Have the teacher initial your work.

Challenge 5

14. If you had derived the formula in Challenge 4 from the beginning, you would not have needed to do all the other work leading up to it. This shows how algebra can serve as a language that reveals the truth about a phenomenon. Pretty cool, huh? Now you have the chance to get in on the ground floor. Use algebra to show how the circumferences of the smaller circles relate to the circumference of the larger circle.

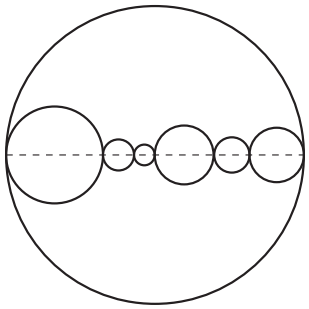
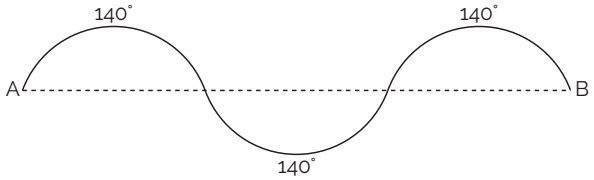


Figure 3: Circles in Circles

Challenge 6

Are you dancing on the desk because of the coolness of your discovery? Have you considered asking the teacher to reward you with...your own plane?

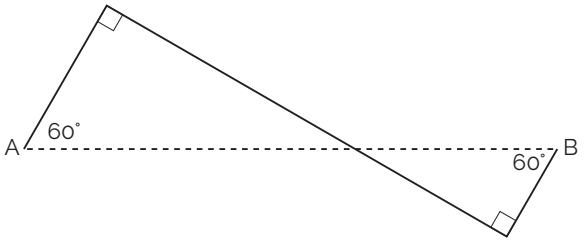
Hold on 'cause we're just getting started. We've been exploring snaking rivers made of semicircles. What if the rivers were made of other shapes?



15. The diagram above shows a river made of three congruent 140° arcs. Choose a value for *AB*, and calculate the length of this river.
16. Sketch an arc-based river using some other number of 140° arcs. Calculate the length of this river.
17. Now derive a generic formula for the length of a river based on *n* 140° arcs of different sizes. (You did this for semicircles in *Challenge 4*.)
18. You used arcs with the same measure. Explain why you would or would not get these same results if you use arcs with different measures.
19. Have the teacher initial your work.

Challenge 7

Let's see what happens when we use rivers that are not based on circles at all. Here's a river made of 30-60-90° triangles.



20. This time let's jump right to the most generic case possible. Derive a formula for calculating the length of a river made from *n* 30-60-90° triangles.
21. Explain why this did or did not turn out just like the results for the *n*-semicircle case.

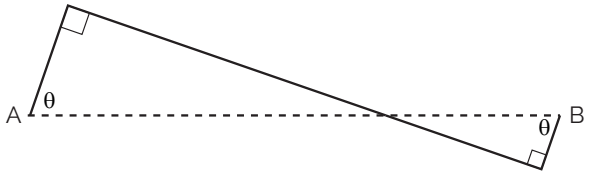
Let's represent the formulas you have been deriving with one generic formula:

$$L = k \cdot AB$$

22. In this formula, *k* represents a constant. Determine and list the values of *k* for the semicircular river, the 140°-arc river, and the 30-60-90° triangle river.
23. Have the teacher initial your work.

Challenge 8

Still want more? How about a river made of right triangles in which one of the non-right angles measures θ? (Pick a value.) If this gives you the same results that you've been getting, you'll have the most generic result possible for right triangles.



24. Derive a formula using trigonometry that you can use to find the length of an *n*-triangle river.
25. Have the teacher initial your work.

Challenge 9

26. Represent your results from *Challenge 8* using a constant, *k* (like you did in 21). Write an expression for *k* in terms of θ.
27. Use your graphing calculator to find the value of θ for which the river is longest. What is the largest possible value for *k*?
28. Have the teacher initial your work.

Challenge 10

Explain how you could avoid this whole project and derive a single result if you view the rivers as being made of similar figures.